Interactions between hadrons are strongly modified near the QCD (tri)critical point

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The QCD (tri)critical point, a genuine second-order phase transition, implies existence of a massless scalar mode, and singular behavior near it. In this work we however focus on the finite region around it, defined by a condition $m_\sigma = 2m_\pi$. We point out that in this region the inter-hadron interaction should be dramatically changed. Light sigma should increase attractive mean field potentials for baryons and non-Goldstone mesons. The same effect can be observed in additional downward shift of the mass of vector mesons ρ, ω, ϕ, accessible via dilepton experiments. For pions we predict that the mean change due to light sigma is in fact a *repulsive* mean potential. The implications of these effects for collective pion and nucleon flows, radial directed and elliptic, are estimated. Finally, we speculate tha unusual behavior of flows observed by NA49 at 40 GeV PbPb collisions at SPS may be explained by location of the critical point region nearby.

I. INTRODUCTION

A. The QCD (tri)critical point

Let me start by briefly reminding well known facts. Ignoring strangeness and starting with massless u,d quark theory, one arrives at the situation in which the phase transition at small baryon chemical potential μ is a second order line, changing to a first order line at the so called *tricritical point*. In this theory it is convenient to look at pions and sigma as one 4-component field ϕ_i and write Landau-Ginzburg potential in terms of its square

$$\Omega = \frac{a}{2}(\phi_i\phi_i) + \frac{b}{4}(\phi_i\phi_i)^2 + \frac{c}{6}(\phi_i\phi_i)^6 \tag{1}$$

The coefficients $a(T,mu), b(T,mu), c(T,mu)$ are some functions. The second order line is defined by zero mass condition $a(T,mu) = 0$: at it all 4 fields π, σ are massless, so the transition belongs to the universality class of the O(4) spin model. At the tricritical point additionally the second equation holds $b(T,mu) = 0$ which together with the first one select one critical point. Here the indices are as in the mean field theory.

In the real world the nonzero quark mass adds term $m\sigma$ (and other odd terms) to this nice symmetric Lagrangian and makes it a bit more compacted. Still straightforward manipulations with this function tell us how all quantities change along the critical line, see the second paper [1]. It makes all fields massive in the vacuum, and transform the second order line to just a crossover (see Fig.1). ri The tricritical point is changed to just a critical point with Ising exponents and just one massless mode, $m_\sigma = 0$. The pions are massive there, and $m_\pi^2 \sim m^{4/5}$ which is only slightly differs from m^1 in G-Oaks-Renner relation in the vacuum. Below we will ignore small correction to it, $\sim (\Lambda_{QCD}/m)^{1/10}$, and assume the pion mass at the critical point is the same as in vacuum. Obviously, there should be a finite region (the shade area in Fig.1 to be refered to as "oval" below) in which $m_\sigma < 2m_\pi$. One consequence of this condition is that $\sigma \to 2\pi$ decay is impossible: but the boundary of the oval will be important for other reasons as well, see section IV.

Stephanov, Rajagopal and myself [1] were the first to propose experimental search for the QCD (t)critical point in heavy ion collisions, by varying the energy of and looking for "non-monotonic" signals. The first specific signal proposed in that paper was an increased *event-by-event fluctuations* near it, reminiscent of critical opalescence well known near other 2-nd order phase transitions.

Although well motivated, the fluctuations is a rather subtle signal, because (as in fact emphasized in original papers on event-by-event fluctuations [2,3]) the experimentally observed fluctuations are most likely to be just equilibrium thermodynamical fluctuations at the thermal freeze-out. Therefore, critical fluctuations can only be seen due to some non-equilibrium "memory effects". Asakawa, Heinz and Muller [4] and Jeon and Koch [5] suggested that large-scale fluctuations of conserved currents (such as electric and baryon charges) are more slow to dissipate, and argued that those may show the "primordial QGP" values, rather than those for hadronic gas at freezeout. Unfortunately further studies (e.g. by Stephanov and myself [6]) have shown that the diffusion from the boundary of experimentally covered begins to be large enough to wipe it out. Experimentally, at any collision energy studied so far, the charge fluctuations seen are close to resonance gas values, while small expected "QGP values" unfortunately are not observed.

The second proposal of [1] was the so called "focusing effects", a particular deformation of adiabatic cooling lines in the vicinity of the critical point. Nonaka and Asakawa [7] have provided a description of this effect in significant detail.

In this paper I would like to reconsider the issue and suggest (hopefully) more robust signals of the critical point, which would not rely on subtle memory effects. One of those is collective flows, which show the effect *accumulated during the whole evolution* along the cooling path, rather than just its end the freezeout. The second, even more direct but more challenging observable, is the invariant mass spectrum of dileptons, some of which produced right when the system's cooling path passes the oval, perhaps even near the critical point.

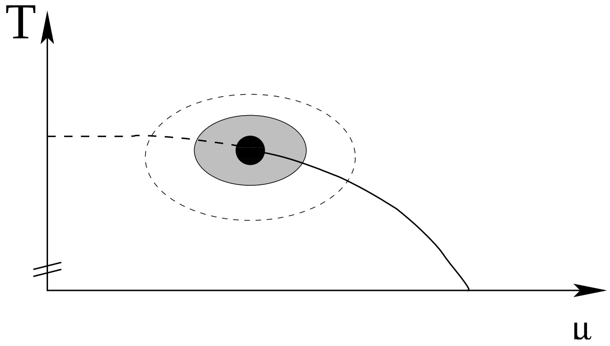

FIG. 1. Schematic view of the critical point, for 2 flavor QCD with nonzero quark masses. The black dot mark the position of the critical point, separating a crossover (the dashed line) from the first order transition (solid line). The shaded oval shows the region where $m_\sigma < 2m_\pi$ and its decay into pions is forbidden, while the wider ellipse corresponds to the region in which it is less than twice the average pion energy,$m_\sigma < 2 < E_\pi(T)$.

The reduction in sigma mass near the whole critical line, and especially at the beginnig of the crossover region $\mu = 0, T = T_c$ were discussed in numerous models since 1960's, see e.g. [8] and references to earlier works therein. But susceptibilities obtained in lattice simulations before, at finite T and *zero* μ, do not show any large peaks corresponding to light sigma* . At the critical point, however, the sigma mass *must* be zero by definition, regardless of the quark masses†. Furthermore, this work was triggered by new lattice data (see next subsection) have now indicated a quite different behavior of these susceptibilities, consistent with quite small sigma masses and consequently large effects at $\mu \approx T_c$.

The discussion of critical properties themselves reduces to the statement that it belongs to the simplest Ising universality class, with standard critical exponents. The issue of dynamical exponents and universality class was discussed by Son and Stephanov [9]: the only relevant conclusion the reader should be reminded now is that the dynamical index z, relating the available time the system spends near the critical point

$$\tau \sim \xi^z \sim m_\sigma^{-z} \tag{2}$$

with the maximal correlation length ξ, is $z \approx 3$. Therefore, in a realistic central collisions of the heaviest nuclei available, the time limits the correlation length and the sigma mass by about

$$\xi < 3\, fm, m_\sigma > 70\, MeV \tag{3}$$

Still, a significant reduction of the sigma mass from its vacuum value, by about an order of magnitude, may be possible. As we argue in this work, even a decrease by about factor 2, bringing us close to “oval” in Fig.1, should result in quite dramatic changes in inter-particle interaction.

B. Recent lattice results

Discussions of the possible search for the critical point in experiment has resulted in a search for it on the lattice as well. Of course this is a very difficult task, as direct simulations at finite μ are impossible due to the notorious sign problem. Two methods pursued are re-weighting along the critical line [10] and Taylor expansion in powers μ/T. (We remind the reader that μ in this work is defined per quark, so $\mu/T = 1$ means the usual baryon number chemical potential to be about 500-600 MeV.)

Let me just focus on one most resent study of the latter type, by Bielefeld-Swansee group [11] which followed such an expansion for the 2-flavor QCD up to terms $O((\mu/T)^6)$. The authors conclude that they don't actually see any direct signatures of the critical point yet, nor in fact any other indications for the presence of the first order transition at larger μ. In the covered domain, namely $\mu/T < 1$, the Taylor series seem to converge reasonably well. This is not surprising, since their quark mass is still quite large compared to physical one, which is suppose to move the critical point toward larger μ. And yet, a closer look at their results show that their numerical data are quite remarkable, they show surprisingly abrupt changes in the system, and some indications are perhaps relevant for the issues discussed in this work.

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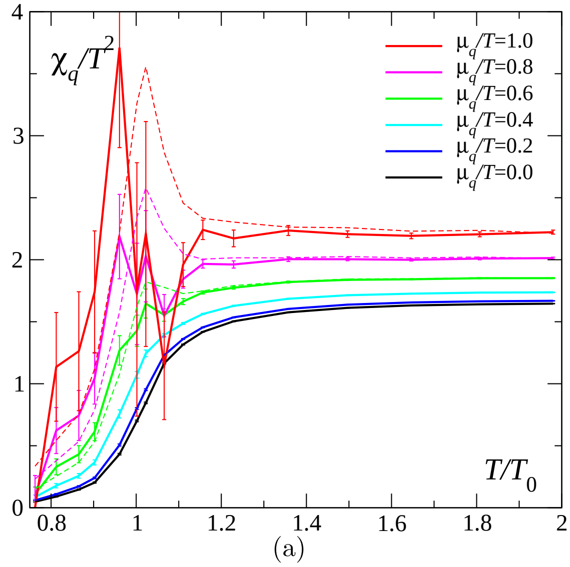
*For the quark mass value used in the calculations, of course.

†Note that although we know from experiment how experimental sigma resonance is seen in pion and kaon scattering, we do not know that for the critical mode. Thus its interaction with strange quarks may be different from that with light ones: the corresponding mixing angle between SU(3) singlet and octet is unknown parameter.



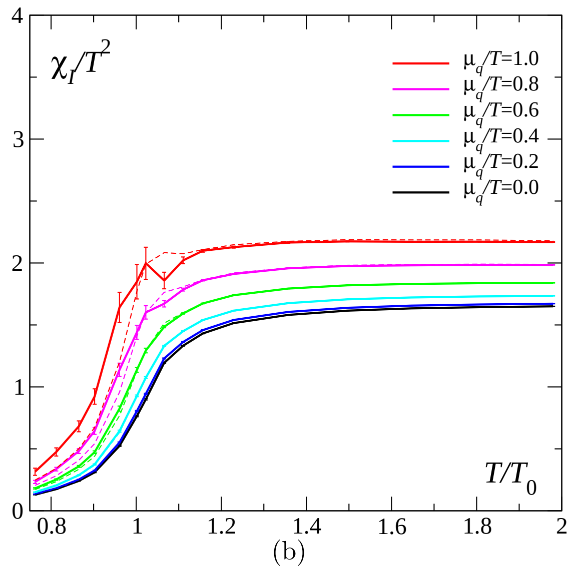


FIG. 2. The quark number susceptibility χ_q/T^2 (left) and isovector susceptibility χ_I/T^2 (right) as functions of T/T_0 for various μ_q/T ranging from $\mu_q/T = 0$ (lowest curve) rising in steps of 0.2 to $\mu_q/T = 1$, calculated from a Taylor series in 6^{th} order. Also shown as dashed lines are results from a 4^{th} order expansion in μ_q/T.

In Fig.2 borrowed from this work we show the normalized quark number susceptibility χ_q/T^2, for the usual (baryon number) chemical potential, as well as for isospin-related one. One can see that, as μ/T grows, the former one develops a sharp peak below T_c, while the isovector susceptibility does not.

Is this peak due to 4-th order 2-loop diagram with a sigma meson exchange, shown in Fig.3(a)? It would show a peak in μ^4 coefficient (as indeed observed on the lattice) and no peak in the isovector case (again, as observed)?

Redlich and Karsch [12] argued that (at least the left-hand side of the peak $T < T_{peak}$) is not due to this diagram, which is suppressed by a need to excite a baryon in each loop and is thus suppressed by extra $\sim exp(-M_N/T) \approx 1/300$. They find it hard to think that even enhancement due to small sigma mass $\sim 1/m_\sigma^2$ of the diagram (a) can beat that. They argue instead that the l.h.s. of the peaks is quite well described by a baryonic resonance gas[‡] and the usual one-loop diagram (b).

If so, why does the signal drop so rapidly at the right-hand side of the peak? One possible answer is *rapid melting of many baryonic states* above the critical line. Another possibility is that the susceptibility is decreasing because (in spite of deconfinement) the baryon mass is larger than in vacuum. In fact, the effective quark mass (defined as half potential between $\bar{q}q$ at large separation) is indeed so large above the critical line, that quark contribution $\sim exp(-M_q^{eff}(T)/T)$ can get even more suppressed than that of the nucleon.

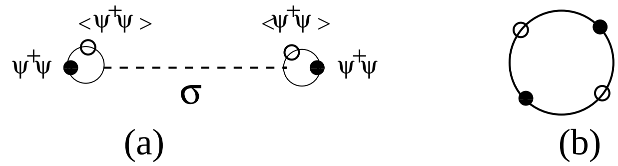


FIG. 3. Two diagrams responsible for 4-th order part of the density-density susceptibility. The black dots stand for the original vector density operators, the open circles are for insertions of the mean vector density of the matter. The solid lines are states with the baryon number, the baryons below the critical T and quarks/diquarks above it. The dashed line represents the σ meson.

Now let us look at the behavior of the chiral condensate and chiral susceptibility $\chi_{\bar{\psi}\psi}$, shown in Fig.4 from the same lattice study. The condensate shows little change, just slight shift downward in the critical T as μ grows, as expected. The susceptibility however seem to have a larger peak: in contrast to plots shown above, now there is no need for extra vector current insertions, so it probably is the (unsuppressed) sigma contribution. Thus, in spite of larger uncertainties and similarity to the plot above, I still think this peak does indeed contain a significant part of a sigma exchange, and therefore the peak is the first manifestation of reduced sigma mass. If the absolute hight of the peak is a measure of $1/m_\sigma^2(\mu)$, one may thing the reduction of the mass itself is by a factor 2 or so. Thus, optimistically, one may think that at least the edge of the "oval" of Fig.1 is more or less reached by those simulations. Needless to say, more work is need to see if this is correct: the simplest of those is to see a correlator rather than integrated susceptibility.

[‡]With masses of mesons and baryons correctly adjusted to quark masses used in the lattice simulations.

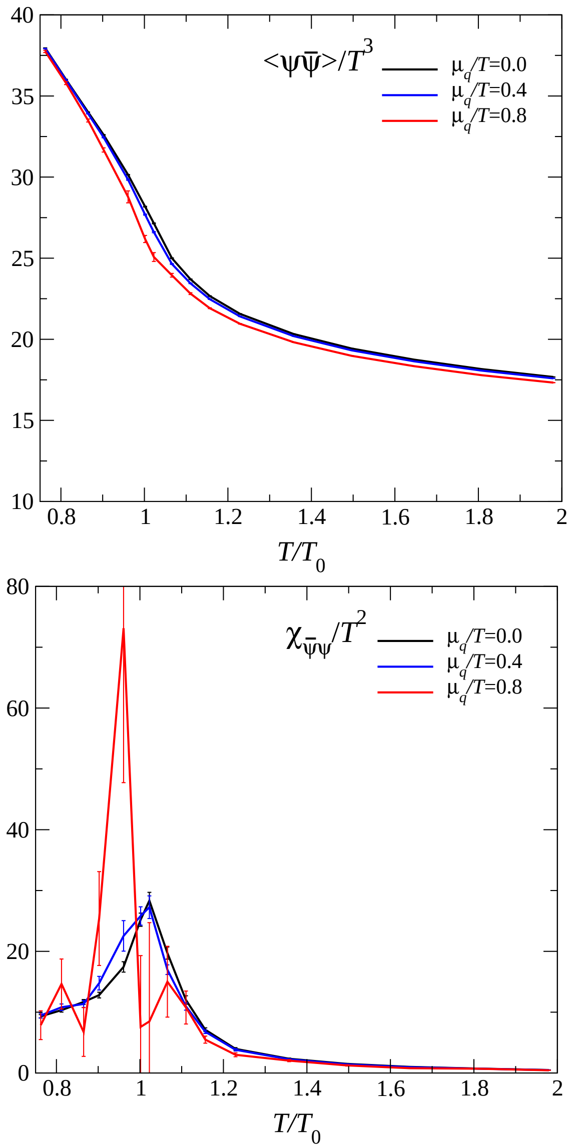


FIG. 4. The chiral condensate $\langle\bar{\psi}\psi\rangle$ (left) and chiral susceptibility $\chi_{\bar{\psi}\psi}$ (right) as a function of T/T_0 for $\mu_q/T = 0,\ 0.4$ and 0.8. The chiral condensate drops with increasing μ_q/T and the peak in $\chi_{\bar{\psi}\psi}$ becomes more pronounced.

Summarizing this subsection, the results of numerical simulations presented in [11] does not found the critical point. Instead, they show a dramatic change of the baryon contributions to thermodynamics, from a resonance gas with unmodified masses, to a completely different regime in QGP. Chiral accessibility possibly show reduction in the sigma mass by a factor of 2 for $\mu \approx .8T_c \approx 130\,MeV$, which however remains rather uncertain.

II. NUCLEAR FORCES

Studies of the NN forces, are at the very basis of nuclear physics, go back by many decades and are discussed in textbooks in detail. We will not discuss spin-spin or spin-orbit parts of it, but focus rather on one model, as simple as possible, for the central forces. A well known Walecka model [13] is sufficient to demonstrate our main points.

Those forces can be viewed as a combination of the sigma exchange[§] and the omega exchange terms

$$V = -\frac{g_s^2}{4\pi}\frac{e^{-rm_\sigma}}{r} + \frac{g_v^2}{4\pi}\frac{e^{-rm_\omega}}{r} \qquad (4)$$

In this simple model $g_s = M_n/f_\pi \approx 10$ and $g_V^2 = 190$ which fits the data well enough.

The most important lesson about nuclear forces we would like to remind the reader is that the nuclear potential is in fact a *highly tuned small difference* of two large terms. Although both terms are comparable to the nucleon mass, the resulting potential has a minimum of only about a percent of the available mass, $V(r_{min})/2M_N \sim O(10^{-2})$. [**] The so called "relativistic systematics", the dependence of nuclear forces on relative motion, reveals this fact well, as the scalar and vector parts change differently under Lorentz transformation. It makes the forces repulsive at semi-relativistic energies, as multiple NN and heavy ion low energy collisions have well documented.

Because of this fine tuning, the nuclear forces are quite sensitive to the 4 parameters of the model, two couplings and two masses[††].

If one simply takes $m_\sigma \to 0$, keeping the other parameters the same, one gets huge attraction $V \sim -1GeV$. If on the other hand the omega mass is also goes to zero, the result is huge repulsion[‡‡] $V \sim +1GeV$. More moderate version of the same exercise (with parameters to be explained later) is demonstrated in Fig.5 .

[§] Strong variation of the phase shift at 400-600 MeV is known for a very long time in $\pi\pi$ scattering, and was also identified in the attractive part of the nuclear forces, see e.g. a review on applications of Walecka model [13]. Whether one would like to call it a resonance or not, the fact remains that it dominates the attractive part of NN interaction, and is thus responsible for nuclear binding and for our very existence.

[**] Of course, in nuclei there is another cancellation between kinetic and potential energy, leading to a typical nuclear physics scale to $1\,MeV \sim 10^{-3}M_N$.

[††] Strong sensitivity of the deuteron binding to m_σ and eventually to the ratio of quark masses to Λ_{QCD} , combined with the Big Bang Nucleosynthesis data, makes the best limits on the cosmological variation of these quantities [14].

[‡‡] Because the omega coupling is larger than the sigma one.

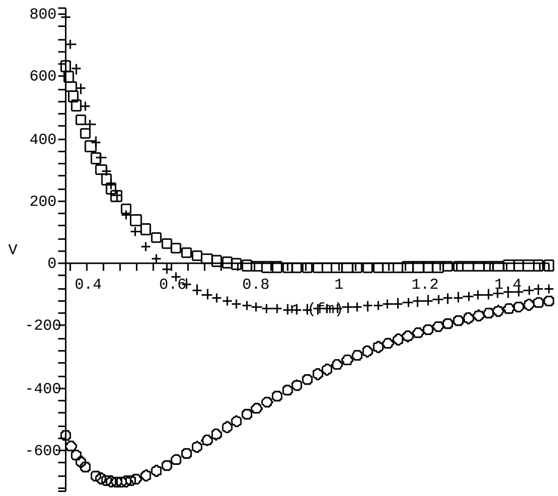


FIG. 5. The NN potentials in the Walecka model, for the following values of the masses: $m_\sigma, m_\omega = 600{,}770$ MeV (black), 280,770 (red), 280,500 (blue)

A nucleon in a homogeneous matter is in a mean field potential which can be easily calculated in this model. The modification of it due to modification of the σ-exchange diagram[§§]

$$\Delta V = -n_s\left(\frac{g_s^{*2}}{m_\sigma^{*2}} - \frac{g_s^2}{m_\sigma^2}\right) \tag{5}$$

where n_s is the total scalar density of matter. Unlike vector density, it includes all quarks and antiquark with the positive sign, so

$$n_s = n_B + n_{\bar{B}} + \frac{2}{3} n_{nGm} \tag{6}$$

where nGm stands for all non-Goldstones mesons. The factor 2/3 corresponds to a simple additive approximation, assuming that the scalar coupling g_s^2 is additive for valence quarks.

III. VECTOR MESONS

The mass shifts for ρ, ω due to sigma exchanges are of the same nature as for baryons, so in the same spirit of additive coupling one expects extra[***] mass shift to be

$$\Delta V_{\rho,\omega} = \frac{2}{3}\Delta V_N = -\frac{2}{3} n_s\left(\frac{g_s^{*2}}{m_\sigma^{*2}} - \frac{g_s^2}{m_\sigma^2}\right) \tag{7}$$

[§§]What exactly is a sigma field and how it is interaction with pions and nucleons is described e.g. in [15]

[***]Which obviously goes on top of other effects, such as resonances and the sigma exchanges with unmodified mass.

These shifts were experimentally seen in heavy ion collisions e.g. in peripheral AuAu collisions at RHIC, by STAR collaboration [23] for few resonances, including vectors ρ and K^*. In the former case the mass shifts is about $\Delta M_\rho = -70\, MeV$, with about half of that estimated to come from the sigma exchange [22]. If so, one may thing that at the edge of the "oval" this term would be 4 times larger, or about $-120\, MeV$. Very close to the critical point the effect is still finite, in spite of zero sigma mass, due to vanishing coupling: this however only happens when sigma size gets really large.

The resonances like ρ seen via $\pi\pi$ channel are of course observed near the kinetic freezeout conditions, that is at rather dilute matter. One may wander if those are not too far from the critical line and critical point, to show any observable effect.

Another well known possibility is observation of vector states via dileptons: in this case they are collected not from freezeout but from all the 4-volume of expanding fireball, with the inside of the "oval" included. Since we expect additional shift of vector mass, the enhancement is supposed to increase roughly as $exp(\Delta M(T)/T) \sim 3$ due to twice lighter sigma.

We do indeed know that NA45 (CERES) collaboration have seen rather large enhancement of small mass dileptons at SPS, especially at 40 GeV. Recent run of NA60 experiment with dimuons is expected to provide additional clarification on this issue. A non-monotonous dependence of the light dilepton continuum on collision energy would be possible indication for the critical point.

IV. THE INTERACTION OF THE PIONS

Unlike baryons and non- Goldstone modes of the theory, the pions cannot simply linearly interact with the sigma field, because any nonzero scalar density would made them massive even in the chiral limit and thus violate the Goldstone theorem. In the original sigma model formulation that is resolved by some cancellation between diagrams. Better solution is to change the scalar field to its "radial" version on the chiral circle (see [15]), after which sigma interacts only with a derivatives of the pion field and preserves the theorem in each vertex.

As in the chiral limit the interaction of low energy pions is described by Weinberg Lagrangian,

$$L_W = \frac{1}{2f_\pi^2}(\partial_\mu U^+)(\partial_\mu U) \tag{8}$$

the corrections due to nonzero scalar density of the medium can simply be absorbed into a modified coupling

$$\frac{1}{f_\pi^2} \to \frac{1}{f_\pi^2} + n_s * G \tag{9}$$

to be substituted into known results about pion gas thermodynamics [17,18] and kinetics [19]. For low-T pion gas the same is true for realistic QCD with massive quarks

and pions, since $O(m)$ corrections to scattering (such as Weinberg scattering lengths) are very small.

A real hadronic matter produced in heavy ion collisions have sufficiently high temperature $T > T_f \sim 100\,MeV$ to excite resonances, which truly dominate the interaction. For pions those are σ and ρ. those provide scattering rate and ImV. The real part of the mean field potential for pions is very small and usually neglected.

The (momentum-dependent) potentials are usually defined as

$$V(p) = (m^2 + p^2 + \Sigma(p))^{1/2} - (m^2 + p^2)^{1/2} \qquad (10)$$

where

$$\Sigma_j(p) = \sum_i \int \frac{d^3k}{(2\pi)^3} \frac{n(\omega_k/T)}{2\omega_k} M_{ij}(k,p) \qquad (11)$$

is the mass operator related to the forward scattering amplitude M. The sum stands for all particle types in matter.

The amplitude can in turn be written as a sum over resonances in each channel, and we are going to focus on σ in $\pi\pi$. For this case the forward scattering amplitude is of standard Breit-Wigner type

$$M_\sigma = -\frac{4\pi E}{q} \frac{\Gamma_{in}}{m_\sigma - E + i\Gamma_{tot}/2} \qquad (12)$$

where $E = \sqrt{s}$ and q are the CM energy and momentum of two colliding pions with momenta $\mathbf{p}, \mathbf{k}$. We used the total width in the form $\Gamma_{tot} = (.4\,GeV)(q/E)$ where q, E are the C.M. momentum and energy per pion: not that it vanishes at q=0, as is needed by the phase space. The $\Gamma_{in} = \Gamma_{tot}/2$.

As it is well known, the real part of the amplitude *changes sign* at the resonance, and the sign means that effective potential obtains positive (*repulsive*) contribution from $\pi\pi$ states above the resonance, $E > m_\sigma$, and negative (*attractive*) from the states below it. These two parts of the integral over k tend to cancel each other. As a result, the interaction between pions is weakly attractive in a pion gas, with a potential strongly dependent on the pion momentum p.

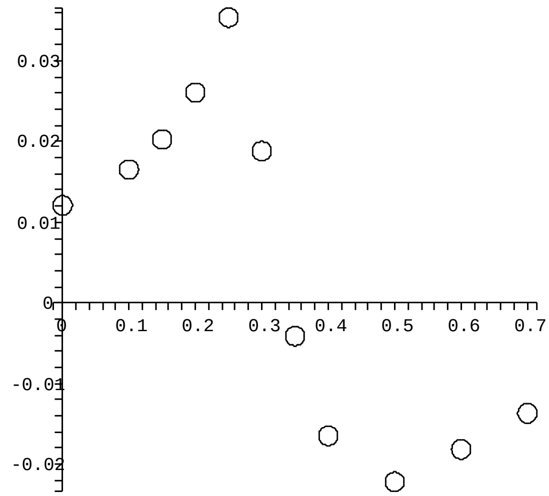


FIG. 6. Effective potential for a pion at rest $Re[V_{eff}(p=0)]$ [GeV] induced by sigma resonance, as a function of the sigma mass M_σ [GeV].

Another (somewhat more intuitive)) picture of pion in a pion gas is that since they spend part of the time rotating around each other in form of resonances, the average pion's velocity is slightly reduced in the pion gas relative to that in the vacuum [20].

However this is going to change near the QCD critical point. Near the dashed line in Fig. 1 the effective pion potential rapidly changes and is getting repulsive inside the solid shaded oval, as seen from Fig.6. For light enough sigma, all colliding pions are always *above* the sigma resonance, and thus the attractive contribution is absent, with only repulsive contribution of sigma resonance remaining. As seen from the figure, it leads to a rapid change of the potential, which is not small. It happens exactly at the boundary of the oval, where $m_\sigma = 2m_\pi$ and the scattering length of two pions at rest goes to infinity since the amplitude is $\sim 1/q$ and the relative momentum $q \to 0$ here. The corresponding $\pi\pi$ cross section at this point is reaching its s-wave unitary limit

$$\sigma \approx \frac{4\pi}{q^2} \qquad (13)$$

This behavior is identical to the so called "Feshbach resonance" at zero energy, which is used in atomic physics for cold trapped atoms. The result is known to be a new "strong coupling" regime of matter in which it shows liquid-like behavior [21].

In summary of this subsection: we predict that close to QCD critical point the pion-pion interactions rapidly change their sign, leading to repulsive mean potential.

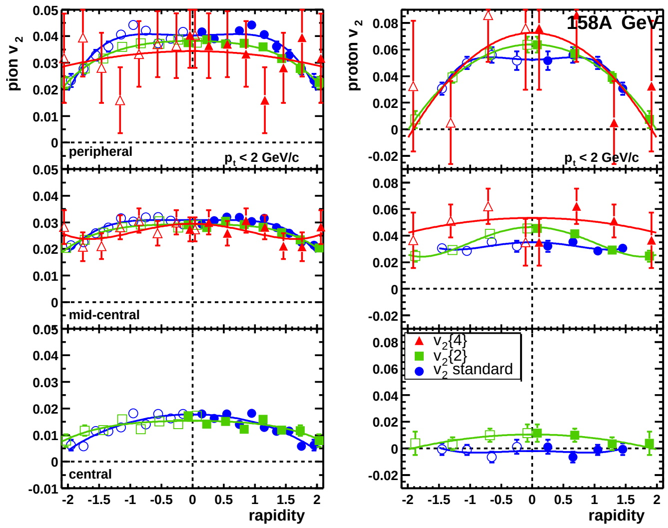


FIG. 7. Elliptic and directed flows of pions and protons versus rapidity at 158 A·GeV Pb+Pb collisions [16] measured for three centrality bins: central (dots), mid-central (squares) and peripheral (triangles). The solid lines are polynomial fits to the data [16].

V. DIRECTED FLOWS

One general difference in the vicinity of the critical point is that near it the hydrodynamical description is to be appended by simultaneous solution for the critical slow mode, the σ field, coupled to matter evolution[†††]. The first pioneering attempt to do so has been done recently by Dumitru, Paech and Stocker [26].

We have argued above that near the critical point the nucleon-nucleon and nucleon-non-Goldstone-meson interaction should gets much more attractive than it is in vacuum. We will now argue that it should affect collective flows of nucleons, reducing their radial and elliptic flows. At the same time, we have argued in the previous chapter that the pion-pion interaction gets more repulsive, so we expect the effect of the opposite sign in the pion flows.

Potentials related to collective sigma field add to pressure effect in defining the flows. ∇p term gets addition, $n_s \nabla \sigma$. The distribution of the mean field sigma is to be found from the distribution of the scalar density via

$$(-\nabla^2 + m_\sigma^2)\sigma(x) = n_s \tag{14}$$

(footnote: why not time derivatives? expansion is still slow) which should be solved together with hydro equations.

Treating its effect perturbatively

$$\frac{\delta v}{v} \approx \frac{\int dt n_s \nabla\sigma}{\int dt \nabla p} \tag{15}$$

where the integrals are over the world line of each matter cell.

Let us make a very simple order-of-magnitude estimate for the additional contribution to the pion flow we expect. The oval of fig.1 projected to a fireball will correspond to also an ellipsoid-like surface. The pions crossing from inside out would experience a kick from the change in potential, repulsive inside and attractive outside. Non-relativistically, an extra velocity this kick will produce is

$$\Delta v_\pi \approx (\frac{2\Delta V_{eff}}{m_{eff}})^{1/2} \approx .5 \tag{16}$$

where the final value in the r.h.s. is obtained by using the change in the potential $\Delta V_{eff} = 40\, MeV$ from Fig.6 and the effective pion mass for motion in one direction $m_{eff}^2 =< p_t^2 > + m_\pi^2 \approx (2T_c)^2$. This should be compared to collective radial velocity of matter at SPS, which also happen to be of the same magnitude, $< v_r > \sim .5$: the conclusion is the two effects are comparable. Thus expected the non-monotonous change in pion radial flow can be quite noticeable. It will be also accompanied in increased elliptic flow of comparable magnitude.

Similar estimate can be made for the nucleon would include the potential change[‡‡‡] $\Delta U \approx .2GeV$ from Fig.5 and $m_{eff}^2 =< p_t^2 > + m_N^2 \approx 1\, GeV$, resulting in

$$\Delta v_N \approx -.5 - .6 \tag{17}$$

Again, this is comparable to overall matter flow: but since the sign is now *negative* the total effect expected is cancellation of flows. Needless to say, any uncertainties in ΔU and schematic estimate like that cannot substitute for a detailed calculation: we how conclude that there is at least a potential for *dramatic decrease* of both radial and elliptic nucleon flows.

A. Flows observed at SPS

Let us now compare these ideas with observations. As it is well known, elliptic flow changes sign inside the AGS energy domain and is quite well seen at its highest energy, 11 GeV/N. The nucleon and pion elliptic flows are also rather large at the highest SPS energy 158 GeV/N, and about double at RHIC.

[†††] This is similar to what is done e.g. for magnetohydrodynamics, where one component of all fields is singled out from the rest of the matter, and its evolution is treated separately, coupled to the stress tensor and hydrodynamics.

[‡‡‡] Note that the usual nuclear potentials would not be enough.

The summary of the 158 GeV/N elliptic flow data from NA49 experiment is shown in Fig.7. The magnitude is enhanced at mid-rapidity, both for pions and nucleons. As is the case in other cases, and the pion and nucleon v_2 are comparable in magnitude.

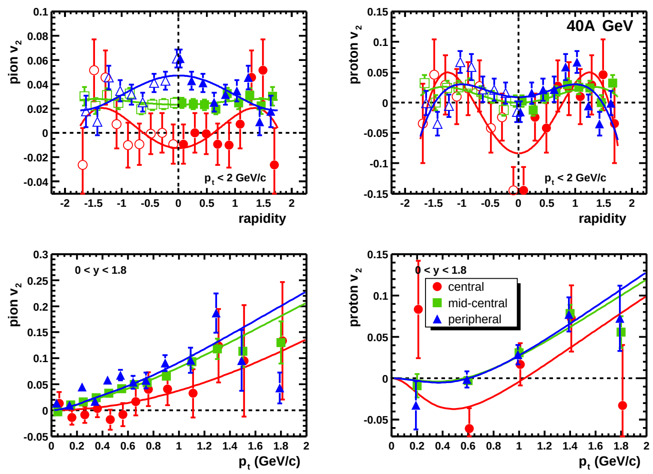


FIG. 8. Elliptic flow v_2 of protons versus rapidity from 40 A·GeV Pb+Pb collisions [16] measured for three centrality bins: central (dots), mid-central (squares) and peripheral (triangles). The solid lines are polynomial fits to the data [16].

It is however not so at 40 GeV/N, as the same NA49 experiment finds Fig.8. Such collapse of flow at mid-rapidity is seen for all centralities and methods. It is furthermore accompanied by a collapse of directed flow v_1: this coincidence tells us that it is unlikely to be an experimental problem and is probably real.

Bratkovskaya et al. [27] suggested that this unusual behavior of the N flow may be a long-expected manifestation of the "softest point". However that contradicts to the fact that elliptic flow of pions does not show any collapse at this energy.

Our current proposal is instead that flows are affected by the approaching critical point. The nucleon flow collapse may be due to attraction effect we discussed, while pions show the opposite potential.

Needless to say, this work is only exploratory in nature, and its suggestion should be investigated quantitatively in future work. In particular, to study flows, one should not only include space-time-dependent sigma field and potentials discussed above, but also cascades which include continuous reactions like $N+\pi- > \Delta- > N+\pi$. Another issue worth considering is the effect of the critical mode on strangeness flows, by $\phi, \Lambda, \Xi, \Omega$.

Finally, in view of all these ideas and observations, from theory, lattice and even hints from data, it seems justified to revisit the low SPS energy region. It can either be done with old SPS detectors (NA49,NA60) in new dedicated runs, or at decelerated RHIC beams, or maybe in future GSI dedicated facility to be built in the next decade.

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